1. Define $f(x) = x^3 - 3x^2 + 3x$.

(a) Estimate the area between the graph of f and the x–axis on the interval $[1,3]$ using a left-hand sum with four rectangles of equal width.

$$\Delta x = \frac{3-1}{4} = 0.5$$

x	$f(x)$
1	1
1.5	1.125
2	2
2.5	4.375
3	9

$$LHS = (1 + 1.125 + 2 + 4.375)(0.5) = 4.25$$

+1 $\Delta x = 0.5$
+1
$f(1) + f(1.5) + f(2) + f(2$

+1 4.25

(b) Is the estimate in part (a) an over-estimate or underestimate of the actual area? Justify your conclusion.

Observe that

$$f'(x) = 3x^2 - 6x + 3$$
$$= 3(x^2 - 2x + 1)$$
$$= 3(x-1)^2$$

Since $f' > 0$, for $x > 1$, the function f is increasing on $(1,3]$. Furthermore, since $f(1) = 1$, f is positive and increasing on $[1,3]$. As a result, each rectangle in the left-hand sum will lie below the graph of f. Thus the area estimate is an underestimate.

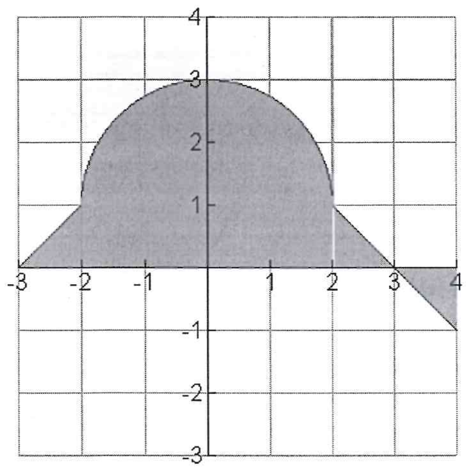
+1 Underestimate
+1 Correct supporting work

(c) Use a definite integral to calculate the exact area between the graph of f and the x–axis on the interval $[1,3]$.

$$\int_1^3 x^3 - 3x^2 + 3x \, dx = \left(\frac{1}{4}x^4 - x^3 + \frac{3}{2}x^2\right)\Bigg|_1^3$$
$$= \frac{1}{4}(3)^4 - (3)^3 + \frac{3}{2}(3)^2 - \left(\frac{1}{4}(1)^4 - (1)^3 + \frac{3}{2}(1)^2\right)$$
$$= 6$$

+1 Correctly integrate f
+1 Correct limits of integration
+1 Apply the Fundamental Theorem of Calculus
+1 Area = 6

4. The graph of a function f is shown in the figure below. It consists of two lines and a semicircle. The regions between the graph of f and the x-axis are shaded.



Editor: $f(x)=\begin{cases} x+3 & -3\le x\le -2 \\ \sqrt{4-x^2}+1 & -2<x<2 \\ -x+3 & 2\le x\le 4 \end{cases}$

(a) Write the definite integral or sum of definite integrals that measures the area of the shaded region. $\int_{-3}^{3} f(x)\,dx+\left\|\int_{3}^{4} f(x)\,dx\right\|$	+1 limits of integration +1 integrand +1 appropriately deal with the canceling of areas
(b) Calculate $\int_0^3 f(x)\,dx$ and $\int_3^4 f(x)\,dx$. The area of a circle of radius 2 is $A=4\pi$. Therefore, the area of a quarter circle of radius 2 is π. The shaded region between $x=0$ and $x=2$ is a quarter circle combined with a rectangle of area 2. Therefore, $\int_0^2 f(x)\,dx=2+\pi$. The area of the triangular region between $x=2$ and $x=3$ is 0.5. Thus $\int_2^3 f(x)\,dx=0.5$. $\int_0^3 f(x)\,dx=2+\pi+0.5\approx 5.641$ The area of the triangular region between $x=3$ and $x=4$ is 0.5. However, since this region is below the x–axis, $\int_3^4 f(x)\,dx=-0.5$.	+1 Area of semi circle is π +1 $\int_0^3 f(x)\,dx=2+\pi+0.5\approx 5.641$ +1 $\int_3^4 f(x)\,dx=-0.5$
(c) Determine the area of the shaded region. $Area=\int_{-3}^{3} f(x)\,dx+\left\|\int_3^4 f(x)\,dx\right\|$ $=2\int_0^3 f(x)\,dx+\left\|\int_3^4 f(x)\,dx\right\|$ $=5+2\pi+0.5\approx 11.783$	+1 $\int_{-3}^{3} f(x)\,dx=5+2\pi$ +1 Appropriately deal with signed area +1 Area = 11.783

7. Define $f(x)=\frac{1}{x}$.

(a) Use the trapezoidal rule with $n=5$ to estimate $\int_1^2 f(x)dx$. $\frac{b-a}{2n}=\frac{2-1}{2(5)}=0.1$ $\int_1^2 f(x)dx \approx 0.1\left(\frac{1}{1}+2\left(\frac{1}{1.2}\right)+2\left(\frac{1}{1.4}\right)+2\left(\frac{1}{1.6}\right)+2\left(\frac{1}{1.8}\right)+\frac{1}{2}\right)$ ≈ 0.696	+1 $\frac{b-a}{2n}=0.1$ +2 Proper use of trapezoidal rule +1 0.696
(b) $\int f(x)=\ln\lvert x\rvert+C$. Use the Fundamental Theorem of Calculus to calculate the <u>exact</u> value of $\int_1^2 f(x)dx$ $\int_1^2 f(x)dx=\ln\lvert 2\rvert-\ln\lvert 1\rvert$ $=\ln 2$	+1 Use Fundamental Theorem of Calculus +1 $\ln 2$ or 0.693 +1 Exact value
(c) Explain why the trapezoidal rule cannot be used to estimate $\int_{-1}^1 f(x)dx$. The function $f(x)=\frac{1}{x}$ is discontinuous at $x=0$. Since the function f is not continuous on $[-1,1]$, the trapezoidal rule may not be used.	+1 f discontinuous at $x=0$ +1 continuity required for trapezoidal rule