

1. Given the region bounded by the graphs of $y = \ln x$, $y = 0$, and $x = e^2$.

a. Use integration by parts to determine the area of the region. $\text{Area} = \int_1^{e^2} \ln x \, dx = x \ln x - \int 1 \, dx, \text{ using integration by parts with } u = \ln x \text{ \& } dv = dx.$ $\text{Area} = (x \ln x - x) \Big _1^{e^2} = e^2 + 1$	+2: integral +1: answer
b. Find the volume of the solid generated by rotating the region about the line $x = e^2$. $\text{Volume} = \pi \int_0^2 (e^2 - e^y)^2 \, dy = \pi \int_0^2 (e^4 - 2e^2 e^y + e^{2y}) \, dy =$ $\pi \left(e^4 y - 2e^2 e^y + \frac{1}{2} e^{2y} \right) \Big _0^2 = \pi \left(\frac{1}{2} e^4 + 2e^2 - \frac{1}{2} \right) = 41.577\pi$	+2: integral +1: limits, constant, answer
c. Find the volume of the solid generated by rotating the region about the horizontal line $y = -2$. $\pi \int_1^{e^2} [(\ln x + 2)^2 - (2)^2] \, dx =$ $\text{Volume} = \pi \int_1^{e^2} [(\ln x)^2 + 4 \ln x] \, dx =$ $\pi \left[x(\ln x)^2 + 2x \ln x - 2x \right] \Big _1^{e^2} =$ $\pi(6e^2 + 2)$	+2: integrand +1: limits, constant, answer

3. Consider the graph of $y = xe^{-x/3}$.

a. Set up the integral used to find the area under the graph of this curve between $x = 0$ and $x = 2$. $\text{Area} = \int_0^2 xe^{-x/3} dx$	+1: integrand +1: limits
b. Identify u and dv for finding the integral using integration by parts. $u = x \quad \text{and} \quad dv = e^{-x/3} dx$	+1: $u = x$ +1: $dv = e^{-x/3} dx$
c. Evaluate the integral using integration by parts. Let $u = x \quad dv = e^{-x/3} dx$ $du = dx \quad v = -3e^{-x/3}$ Then, $\int_0^2 xe^{-x/3} dx = -3xe^{-x/3} - \int -3e^{-x/3} dx =$ $\left[-3xe^{-x/3} - 9e^{-x/3} \right]_0^2 = -15e^{-2/3} + 9 \approx 1.299$	+2: $uv - \int v du$ +2: antiderivatives +1: answer