2. Given the region bounded by the graphs of $y = e^{-x^3}$, $x = 0$, $y = 0$, and $x = b$ $(b > 0)$.

a. Find the area of the region if $b = 1$. Area = $\int_0^1 e^{-x^3}\,dx = .8075$	+2: integral +1: answer
b. Find the volume of the solid generated if the region is rotated about the x-axis and $b = 1$. Volume = $\pi \int_0^1 \left(e^{-x^3}\right)^2 dx = \pi \int_0^1 e^{-2x^3}\,dx = 0.6907\pi \approx 2.170$	+2: integrand +1: limits, constant, answer
c. Find the volume of the solid generated when the region is rotated about the x-axis and $b = 10$. Volume = $\pi \int_0^{10} \left(e^{-x^3}\right)^2 dx = \pi \int_0^{10} e^{-2x^3}\,dx = .7088\pi \approx 2.227$	+2: integrand +1: limits, constant, answer

4. Consider the graph of $y = \ln 2x$.

a. Set up the integral used to find the area under the graph of this curve between $x = 1$ and $x = 3$. Area = $\int_1^3 \ln 2x\, dx$	+1: integrand +1: limits
b. Identify u and dv for finding the integral using integration by parts. $u = \ln 2x$ and $dv = dx$	+1: $u = \ln 2x$ +1: $dv = dx$
c. Evaluate the integral using integration by parts. Let $u = \ln 2x \quad dv = dx$ $du = \frac{1}{2x} 2dx \quad v = x$ Then, $\int_1^3 \ln 2x dx = x \ln 2x - \int x \cdot \frac{1}{x} dx \Big\|_1^3$ $= [x \ln 2x - x]_1^3 = 3\ln 6 - \ln 2 - 2 \approx 2.682$	+2: $uv - \int v du$ +2: antiderivatives +1: answer

7. Consider the function $h(x) = \frac{x - \cos x}{x}$.

a. Can you use L'Hopital's Rule to find $\lim_{x\to\infty} h(x)$? Explain your reasoning. Yes; $\lim_{x\to\infty} h(x)$ is of the form $\frac{\infty}{\infty}$. Even though cosine oscillates, the x term dominates and the numerator goes to infinity.	+1: yes +2: explanation
b. Find $\lim_{x\to\infty} h(x)$ analytically. $\lim_{x\to\infty} \frac{x - \cos x}{x} = \lim_{x\to\infty}\left(\frac{x}{x} - \frac{\cos x}{x}\right) =$ $\lim_{x\to\infty} 1 - \lim_{x\to\infty} \frac{\cos x}{x} = 1 - 0 = 1$	+2: separation into fractions +1: answer
c. Set up the integral needed to find the area under the graph of $h(x)$ between $x = 1$ and $x = \pi$ (above the x-axis). Area = $\int_1^{\pi} \frac{x - \cos x}{x} dx$	+2: integrand +1: limits

8. Consider the function $g(x) = \dfrac{e^x - 1}{x^2}$.

a. Can you use L'Hopital's Rule to find $\lim_{x \to 0^+} g(x)$? Explain your reasoning. Yes; $\lim_{x \to 0^+} g(x)$ is of the form $\dfrac{0}{0}$.	+1: yes +2: explanation
b. Find $\lim_{x \to 0^+} g(x)$ analytically. $\lim_{x \to 0^+} \dfrac{e^x - 1}{x^2} = \lim_{x \to 0^+} \dfrac{e^x}{2x} = \dfrac{1}{0}$. This limit does not exist.	+2: Uses L'Hopital's rule correctly +1: answer
c. Describe the type of indeterminate form involved in directly evaluating $\lim_{x \to \infty} x^2 e^{-x}$. Evaluate this limit using L'Hopital's Rule. The indeterminate form is the form $\infty \cdot 0$. $\lim_{x \to \infty} x^2 e^{-x} = \lim_{x \to \infty} \dfrac{x^2}{e^x} = \lim_{x \to \infty} \dfrac{2x}{e^x} = \lim_{x \to \infty} \dfrac{2}{e^x} = 0$	+1: indeterminate form +2: answer

9. Consider the function $f(x) = \frac{2}{x^3}$.

a. The evaluation of $\int_{-1}^{1} \frac{2}{x^3}\,dx = 0$ is incorrect. Explain why. $\frac{2}{x^3}$ has an infinite discontinuity at $x = 0$.	+2: explanation
b. Determine whether this improper integral converges or diverges. Show your work to justify your conclusion. $\int_{-1}^{1} \frac{2}{x^3}\,dx = \lim_{t\to 0^-} \int_{-1}^{t} \frac{2}{x^3}\,dx + \lim_{s\to 0^+} \int_{s}^{1} \frac{2}{x^3}\,dx =$ $\lim_{t\to 0^-} \left.\frac{-1}{x^2}\right\|_{-1}^{t} + \lim_{s\to 0^+} \left.\frac{-1}{x^2}\right\|_{s}^{1}$ Therefore, this improper integral diverges.	+1: uses limits +1: antiderivative +1: diverges
c. Evaluate $\int_{1}^{\infty} f(x)\,dx$ analytically. $\int_{1}^{\infty} f(x)\,dx = \lim_{t\to\infty} \int_{1}^{t} \frac{2}{x^3}\,dx = \lim_{t\to\infty} \left.\frac{-1}{x^2}\right\|_{1}^{t} = 1$	+2: uses limit +1: antiderivative +1: answer