Key

AP Calculus BC **Polar Functions**

1. Find the slope of the curve $r = 2\cos(4\theta)$.

$$\frac{\cos(4\theta)\cos\theta - 4\sin(4\theta)\sin\theta}{-\cos(4\theta)\sin\theta - 4\sin(4\theta)\cos\theta}$$

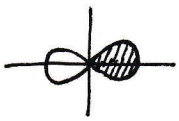
2. Find the slope of the curve $r = 2 - 3\sin\theta$ at $(2, \pi)$.

$$\left.\frac{(2-3\sin\theta)\cos\theta - 3\cos\theta(\sin\theta)}{-(2-3\sin\theta)\sin\theta - 3\cos\theta(\cos\theta)}\right|_{(2,\pi)} = \frac{2}{3}$$

3. Find the area inside the limaçon $r = 4 + 2\cos\theta$.

$$\frac{1}{2}\int_0^{2\pi}(4+2\cos\theta)^2\,d\theta = 18\pi \approx 56.549$$

4. Find the area inside one loop of the lemniscate $r^2 = 4\cos(2\theta)$.

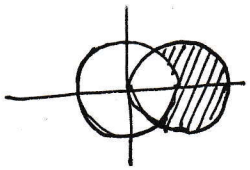


FIND THIS AREA AND DOUBLE IT.

$\alpha = 0$
$\beta = \frac{\pi}{4}$

$$2\left[\frac{1}{2}\int_0^{\frac{\pi}{4}}(4\cos(2\theta))\,d\theta\right] = 2$$

5. Find the area inside $r = 2\cos\theta$ and outside $r = 1$.



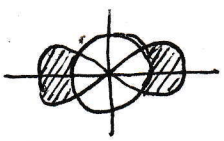
$2\cos\theta = 1$
$\cos\theta = \frac{1}{2}$
$\theta = \frac{\pi}{3}$

AGAIN TOP ½ AREA WILL BE DOUBLED

$$2\left[\frac{1}{2}\int_0^{\frac{\pi}{3}}\left[(2\cos\theta)^2 - (1)^2\right]d\theta\right]$$

$$\frac{\sqrt{3}}{2} + \frac{\pi}{3} \approx 1.913$$

6. Find the area inside the lemniscates $r^2 = 6\cos(2\theta)$ and outside the circle $r = \sqrt{3}$.



$6\cos(2\theta) = 3$
$\cos(2\theta) = \frac{1}{2}$
$\because \cos\theta = \frac{1}{2}$ $\cos(2\theta) = \frac{1}{2}$
$\theta = \frac{\pi}{3}$ $\theta = \frac{\pi}{6}$

TAKE TOP RIGHT AREA AND MULTIPLY BY FOUR

$$4\left[\tfrac{1}{2}\int_0^{\frac{\pi}{6}}\left[(6\cos(2\theta)) - (\sqrt{3})^2\right]d\theta\right]$$

$$\approx 2.055$$