

p.205 $\lim$ is a horizontal asymptote.

(14) a) $\lim_{x \rightarrow \infty} \frac{-4x^2 + 2x - 5}{x} \approx \frac{-4x^2}{x} = -\infty$

Because the exponent on top is larger as x goes to infinity, y grows w/o bound

b) $\lim_{x \rightarrow \infty} \frac{-4x^2 + 2x - 5}{x^2} \approx \frac{-4x^2}{x^2} = \frac{-4}{1} = -4$

Because the top and bottom have the same exponent, you use the coefficients

c) $\lim_{x \rightarrow \infty} \frac{-4x^2 + 2x - 5}{x^3} \approx \frac{-4x^2}{x^3} = 0$

Because the exponent on bottom is larger, as x goes to infinity, y gets smaller

(16) a) $\lim_{x \rightarrow \infty} \frac{3 - 2x}{3x^3 - 1} \approx \frac{-2x + 3}{3x^3} = 0$ (larger exp in bottom)

b) $\lim_{x \rightarrow \infty} \frac{3 - 2x}{3x - 1} \approx \frac{-2x + 3}{3x - 1} = -\frac{2}{3}$ (same exp., use coefficient)

c) $\lim_{x \rightarrow \infty} \frac{3 - 2x^2}{3x - 1} \approx \frac{-2x^2 + 3}{3x - 1} = -\infty$ (larger exp on top)

(20) $\lim_{x \rightarrow \infty} \frac{5}{x} - \frac{x}{3} = \frac{5}{\infty} - \frac{\infty}{3} \rightarrow \text{goes to } 0 - \text{goes to } \infty \approx 0 + \infty = \infty$

(28) $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + 1}} \approx \frac{x}{\sqrt{x^2}} \approx \frac{x}{|x|} = \frac{\infty}{|\infty|} = 1$

(30) $\lim_{x \rightarrow \infty} \frac{-3x + 1}{\sqrt{x^2 + 1}} \approx \frac{-3x}{\sqrt{x^2}} \approx \frac{-3x}{|x|} = \frac{-3\infty}{|\infty|} = -3$

$$(32) \lim_{x \rightarrow -\infty} \frac{\sqrt{x^4-1}}{x^3-1} \approx \frac{\sqrt{x^4}}{x^3} \approx \frac{x^2}{x^3} = 0 \quad (\text{larger exp on bottom})$$

$$(34) \lim_{x \rightarrow -\infty} \frac{2x}{(x^2+1)^{1/3}} \approx \frac{2x}{(x^2)^{1/3}} \approx \frac{2x}{x^{2/3}} \approx \frac{2x}{x^2} = 0$$

$$(36) \lim_{x \rightarrow \infty} \cos \frac{1}{x} \approx \cos 0 = 1$$

$\infty \rightarrow 0$

$$(38) \lim_{x \rightarrow \infty} \frac{x - \cos x}{x} = \frac{x}{x} - \frac{\cos x}{x} = 1 - \frac{\cos x}{\infty} = 1$$

(40) Find the horizontal asymptotes using a calculator
 $f(x) = \frac{|3x+2|}{x-2}$

$$\lim_{x \rightarrow \infty} \frac{|3x|}{x} = 3 \quad \lim_{x \rightarrow -\infty} \frac{|3(-\infty)|}{-\infty} = -3$$

$y = 3$ as $x \rightarrow \infty$ $y = -3$ as $x \rightarrow -\infty$

$$(86) f(x) = \frac{-x^3 - 2x^2 + 2}{2x^2} \quad g(x) = -\frac{1}{2}x + 1 - \frac{1}{x^2}$$

b) Verify algebraically that f and g are the same

$$\begin{aligned} f(x) &= \left(\frac{-x^3}{2x^2} - \frac{2x^2}{2x^2} + \frac{2}{2x^2} \right) \\ &= \left(-\frac{x}{2} - 1 + \frac{1}{x^2} \right) \\ &= -\frac{x}{2} + 1 - \frac{1}{x^2} \end{aligned}$$

