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Find an EQ of the tangent line at (x,y)

(40) $y^2(x^2+y^2)=2x^2$ at (1,1)

$x^2y^2+y^4=2x^2$

$2xy^2+x^2 \cdot 2y\frac{dy}{dx}+4y\frac{dy}{dx}=4x$

$2+2\frac{dy}{dx}+4\frac{dy}{dx}=4$

$6\frac{dy}{dx}=2$

$\frac{dy}{dx}=1/3$

$y-1=1/3(x-1)$

$y-1=1/3x-1/3$

$y=1/3x+2/3$

(47) Find the second derivative

$x^2-y^2=36$

$2x-2y\frac{dy}{dx}=0$

$2x=2y\frac{dy}{dx}$

$\frac{x}{y}=\frac{dy}{dx}$

sub

$\frac{y(1)-x(\frac{dy}{dx})}{y^2}$

$\frac{y-x(\frac{x}{y})}{y^2}$

$\frac{y-\frac{x^2}{y}}{y^2}$

$\frac{\frac{y^2-x^2}{y}}{y^2}$

sub

$\frac{y^2-x^2}{y^3}$

$\frac{-36}{y^3}$

(50) $y^2=10x$

$2y\frac{dy}{dx}=10$

$\frac{dy}{dx}=\frac{10}{2y}$

$\frac{dy}{dx}=\frac{5}{y}$

$\frac{2y(0)-10(2)\frac{dy}{dx}}{4y^2}$

$\frac{-20(5/y)}{4y^2}$

$\frac{\frac{-100}{y}}{4y^2}=\frac{-100}{4y^3}=\frac{-25}{y^3}$

(54) Find the tangent line and normal line.

$x^2+y^2=36$ at $(5,\sqrt{11})$

$2x+2y\frac{dy}{dx}=0$

$2y\frac{dy}{dx}=-2x$

$\frac{dy}{dx}=-\frac{x}{y}=\frac{-5}{\sqrt{11}}$

Tangent: $y-\sqrt{11}=\frac{-5}{\sqrt{11}}(x-5)$

Normal: $y-\sqrt{11}=\frac{\sqrt{11}}{5}(x-5)$

(58) Find the points where the graph has a vertical or horizontal tangent line.

$4x^2+y^2-8x+4y+4=0$

$8x+2y\frac{dy}{dx}-8+4\frac{dy}{dx}=0$

$\frac{dy}{dx}(2y+4)=8-8x$

$\frac{dy}{dx}=\frac{8-8x}{2y+4}=\frac{\text{rise}}{\text{run}}$

Zero rise = Horizontal

$8-8x=0$

$x=1$

$4+y^2-8+4y+4=0$

$y^2+4y=0$

$y(y+4)=0$

$y=0$ $y=-4$

$(1,0)$ $(1,-4)$

Zero run = Vertical

$2y+4=0$

$y=-2$

$4x^2+4-8x-8+4=0$

$4x^2-8x=0$

$4x(x-2)=0$

$x=0$ $x=2$

$(0,-2)$ $(2,-2)$